

FOOTPRINTS OF HIGHER-DIMENSIONAL DECAYING BLACK HOLES

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Compiled March 2, 2013

We review the current results for the emission of Hawking radiation by a higher-dimensional black hole during the Schwarzschild and the spin-down phases. We discuss particularly the role of the angular variation of the emitted radiation on the brane during the latter phase, the radiation spectra for gravitons in the bulk, and the effect of the mass of the emitted particles in determining the bulk-to-brane energy balance.

Key words: Black Holes, Extra Dimensions, Hawking Radiation.

1. INTRODUCTION

The new theories [1–3] postulating the existence of additional spacelike dimensions in nature implemented also a new fundamental scale of gravity which could be considerably lower than the 4-dimensional one. This quickly gave rise to the idea of the manifestation of strong-gravity phenomena at high-energy particle collisions, among them the potential creation of black holes [4–6].

The properties of these miniature black holes have been exhaustively studied [7, 8] including the emission of Hawking radiation which will be the main direct observable effect associated with them. The latter effect is expected to take place during the two intermediate phases in the life of the black hole, the spin-down and the Schwarzschild phase [6]. Whereas the study of the spherically-symmetric Schwarzschild phase is now complete with the radiation spectra having all been derived and the question of the bulk-to-brane energy balance been addressed, these same questions remain open in the case of the axially-symmetric spin-down phase. In the case of the emission along our brane, the radiation spectra for all Standard Model particles have been indeed derived, nevertheless, these cannot currently lead to the determination of both the angular-momentum of the black hole and the number of extra dimensions as was initially hoped. In addition, the derivation of the radiation spectra for the emission of gravitons in the bulk is still in progress, and the question of whether the black hole emits most of its energy in the bulk or on the brane during its spin-down phase remains unanswered.

In this talk, we will review what is known for the emission of Hawking radia-

tion on the brane by a black hole during the Schwarzschild and the spin-down phases - we will particularly address the problem of extracting information on the topological parameters of the theory from the brane radiation spectra, and propose a solution related to the angular variation of the emitted radiation. Then, we will discuss the question of the bulk-to-brane energy balance, the role of the emission of gravitons and of the mass of the emitted particles in determining the dominant channel.

2. CREATION AND PROPERTIES OF HIGHER-DIMENSIONAL BLACK HOLES

Throughout this talk, we will adopt the scenario of Large Extra Dimensions [1] according to which a number n of additional, flat, spacelike dimensions exist in nature apart from the usual three. For simplicity, these extra dimensions are assumed to have the same size \mathcal{R} and to make up a compact space of volume $V \sim \mathcal{R}^n$. Our $(3+1)$ -dimensional world is a hypersurface, a *3-brane*, embedded in the $(4+n)$ -dimensional spacetime, the *bulk*. On our brane the usual Standard-Model particle physics holds, with the gravitational interactions becoming strong only at the Planck scale $M_P = 10^{19}$ GeV. Unlike the Standard-Model particles that are localised on the brane, gravitons are allowed to propagate in the whole of spacetime and mediate a higher-dimensional gravitational force – this means that if two test particles with masses m_1 and m_2 are brought at a distance $r \ll \mathcal{R}$, the corresponding force will vary with r as $1/r^{n+2}$. Moreover, one may further assume that the magnitude of the gravitational force will be determined by a higher-dimensional Newton's constant G_D ; demanding that, in the limit where r increases at values much larger than \mathcal{R} , contact should be made with the 4-dimensional expression, it is found that [1]

$$G_D \simeq G_4 \mathcal{R}^n. \quad (1)$$

If we further define $G_D \sim 1/M_*^{n+2}$, and use the relation $G_4 \sim 1/M_P^2$, the above formula takes the form

$$M_P^2 \simeq \mathcal{R}^n M_*^{2+n}, \quad (2)$$

where M_* is the energy scale where gravitational forces become strong in the context of the higher-dimensional theory. If, unlike string theory, we assume that the size of the extra compact dimensions is much larger than the Planck length, then M_* can be considerably smaller than M_P .

The most optimistic scenario assumes that M_* can be as low as a few TeV, an energy scale that is accessible at current particle-physics collision experiments. Therefore, if the center-of-mass energy E exceeds M_* , then, strong gravity phenomena may become manifest. One of these phenomena is the creation of a black hole [4] during the collision of two ordinary brane-localised particles with impact parameter b : if $b < r_H(E)$, where $r_H(E)$ is the Schwarzschild radius that corresponds to E , then a black hole will be formed according to the Hoop Conjecture [9, 10].

The produced black hole, being a gravitational object, will be generically higher-dimensional and will extend both along and off the brane. In the limit $r_H \ll \mathcal{R}$, the black hole lives in a $(4+n)$ -dimensional spacetime where no distinction can be made between the n compact spacelike dimensions and the three infinite-sized ones. The simplest prototype that we may adopt to describe such a higher-dimensional black hole is the spherically-symmetric Schwarzschild-Tangherlini one [11, 12]

$$ds^2 = - \left[1 - \left(\frac{r_H}{r} \right)^{n+1} \right] dt^2 + \left[1 - \left(\frac{r_H}{r} \right)^{n+1} \right]^{-1} dr^2 + r^2 d\Omega_{2+n}^2, \quad (3)$$

where $d\Omega_{2+n}^2$ is the line-element of a $(2+n)$ -dimensional unit sphere

$$d\Omega_{2+n}^2 = d\theta_{n+1}^2 + \sin^2 \theta_{n+1} \left(d\theta_n^2 + \sin^2 \theta_n \left(\dots + \sin^2 \theta_2 (d\theta_1^2 + \sin^2 \theta_1 d\varphi^2) \dots \right) \right). \quad (4)$$

If we apply the Gauss law in $D = 4 + n$ dimensions, we find the following relation between the black-hole horizon radius and its mass [12]

$$r_H = \frac{1}{M_*} \left(\frac{M_{BH}}{M_*} \right)^{\frac{1}{n+1}} \left(\frac{8\Gamma(\frac{n+3}{2})}{(n+2)\sqrt{\pi}^{(n+1)}} \right)^{1/(n+1)}. \quad (5)$$

The above formula is a generalised one holding for a higher-dimensional black hole – the well-known linear relation between the mass and the horizon radius holding in 4 dimensions arises if we set $n = 0$ in the above expression, and also replace the fundamental Planck scale M_* with the 4-dimensional one M_P .

Although a Quantum Theory of gravity would be the natural framework in which such high-energy particle collisions and their strong-gravity consequences should be studied, one may draw valuable conclusions by using purely classical arguments. Therefore, one may assert that a black hole could be created if the Compton wavelength $\lambda_C = 4\pi/E$ of the colliding particle of energy $E/2$ lies within the corresponding Schwarzschild radius $r_H(E)$ [13]. By using the above-derived expression for the horizon radius (5), we may write this creation condition in the form

$$\frac{4\pi}{E} < \frac{1}{M_*} \left(\frac{E}{M_*} \right)^{\frac{1}{n+1}} \left(\frac{8\Gamma(\frac{n+3}{2})}{(n+2)\sqrt{\pi}^{(n+1)}} \right)^{1/(n+1)}. \quad (6)$$

The above formula may be solved for the ratio $x_{min} = E/M_*$ that determines the value of the center-of-mass energy of the experiment necessary for the creation of the black hole. We find that, for $n = 2 - 7$, the minimum energy of the collision should be $E = (8.0 - 11.2) M_*$, respectively [13]. As the maximum center-of-mass energy that will be achieved at the Large Hadron Collider at CERN is 14 TeV, we immediately see that a window of energy of maximum width of 6 TeV, in the optimistic case where $M_* = 1 \text{ TeV}$, will be open for the search of miniature black holes.

In order for the produced black hole to be considered as a classical black hole, its mass should be at least a few times larger than the fundamental scale of gravity. As an indicative case, we may consider $M_{BH} = 5 \text{ TeV}$ for $M_* = 1 \text{ TeV}$. Then, Eq. (5) can give us the value of the horizon radius: as n varies from 1 to 7, we obtain $r_H = (4.06 - 1.99) 10^{-4} \text{ fm}$ [7]. Note that the presence of the new scale of gravity M_* in Eq. (5) ensures that the horizon radius increases by more than 30 orders of magnitude compared to its four-dimensional value for the same black-hole mass.

A property of the black hole, that may be associated with observable signatures of their creation, is its temperature. This is defined in terms of the surface gravity of the black hole, and for the spherically-symmetric case of Eq. (3) is given by

$$T_H = \frac{k}{2\pi} = \frac{(n+1)}{4\pi r_H}. \quad (7)$$

By using the above formula and the corresponding values of r_H , we find that the temperature ranges from 77 to 629 GeV for $n = 1 - 7$, respectively [7]. The value of the temperature defines the peak of the radiation spectra associated with the Hawking process, i.e. the emission of elementary particles by the black hole, and that clearly lies in a regime accessible by present-day detection techniques.

As is well-known, no particle can escape through the horizon of the black hole. Nevertheless, when a virtual pair of particles is produced outside the horizon of the black hole and the antiparticle falls inside the black hole, then the particle is free to propagate towards infinity. The particles that reach asymptotic infinity make up the Hawking radiation which is characterized by a thermal spectrum [14, 15]

$$\frac{dE(\omega)}{dt} = \int \frac{|\mathcal{A}(\omega)|^2 \omega}{\exp(\omega/T_H) \mp 1} \frac{d\omega}{(2\pi)}. \quad (8)$$

In the above, ω is the energy of the emitted particles and ± 1 a statistics factor for fermions and bosons, respectively. The factor $|\mathcal{A}(\omega)|^2$ is the Absorption Probability (or, *greybody factor*) and determines the number of particles that will escape the strong gravitational field of the black hole to reach infinity.

If we assume that the evolution of a higher-dimensional black hole is similar to the one of its 4-dimensional analog, we expect the emission of Hawking radiation to be realised during the two intermediate stages of the life of a black hole, the axially-symmetric *spin-down* phase and the spherically-symmetric *Schwarzschild* one [6]. The spin-down phase emerges after the *balding* phase, during which the black hole forms and sheds all quantum and classical conserved charges apart from the ones dictated by the no-hair theorem of General Relativity. The *Planck* phase comes after the emission of Hawking radiation has reduced the black-hole mass at the level of M_* : this quantum object may either evaporate completely, having a lifetime of the order of $\tau = 10^{-26} \text{ sec}$ [16], or reduce to a quantum remnant.

3. DECAY OF HIGHER-DIMENSIONAL BLACK HOLES ON THE BRANE

The simplest phase in the life of a black hole is the one when it is characterised by spherical symmetry. This phase follows when the black hole has emitted all, or almost all, of its angular momentum. The gravitational background around such a black hole is given by Eq. (3). Since we are ourselves observers restricted to live on the brane, we are primarily interested in the emission of particles by the black hole on our brane. To study this effect, we need to derive the equation of motion of particles with arbitrary spin in the gravitational background that is induced by the black hole on the brane. The latter is found by setting the values of all additional θ_i coordinates, with $i = 2, \dots, n+1$, to $\frac{\pi}{2}$ in Eq. (3). Then, the brane line-element takes the form

$$ds_4^2 = - \left[1 - \left(\frac{r_H}{r} \right)^{n+1} \right] dt^2 + \left[1 - \left(\frac{r_H}{r} \right)^{n+1} \right]^{-1} dr^2 + r^2 d\Omega_2^2. \quad (9)$$

The equations of motion of Standard-Model-like fields with spin $s = 0, 1/2, 1$ can be put in the form of a “master” equation [7, 17, 18] by using the Newman-Penrose method [19, 20] and a factorized ansatz for the wavefunction of the field

$$\Psi_s = e^{-i\omega t} e^{im\varphi} \Delta^{-s} R_s(r) S_{sl}^m(\theta). \quad (10)$$

Then, the general field equation reduces to two decoupled equations, one for the radial function $R_s(r)$ and one for the spin-weighted spherical harmonics $S_{sl}^m(\theta)$ [21]. The radial equation has the form

$$\Delta^s \frac{d}{dr} \left(\Delta^{1-s} \frac{dR_s}{dr} \right) + \left[\frac{\omega^2 r^2}{h} + 2i\omega sr - \frac{is\omega r^2 h'}{h} - \lambda_{sl} \right] R_s(r) = 0, \quad (11)$$

where $\Delta \equiv r^2 h \equiv r^2 \left[1 - \left(\frac{r_H}{r} \right)^{n+1} \right]$ – note that, through the non-trivial n -dependence of the metric function, the radial equation of motion of a brane-localised field will depend on the number of transverse-to-the-brane extra spacelike dimensions.

The angular part of the master equation has the well-known form of the differential equation satisfied by the spin-weighted spherical harmonics

$$\frac{1}{\sin \theta} \frac{d}{d\theta} \left(\sin \theta \frac{dS_{sl}^m}{d\theta} \right) + \left[-\frac{2ms \cot \theta}{\sin \theta} - \frac{m^2}{\sin^2 \theta} + s - s^2 \cot^2 \theta + \lambda_{sl} \right] S_{sl}^m(\theta) = 0, \quad (12)$$

where $\lambda_{sl} = l(l+1) - s(s-1)$. Due to the spherical symmetry of the gravitational background, the emitted radiation from the black hole will be evenly distributed over a 4π solid angle, and thus the angular equation offers no new information.

We thus turn our attention to the radial part of the equation, and observe that its solution for the radial function $R_s(r)$ determines the Absorption Probability through

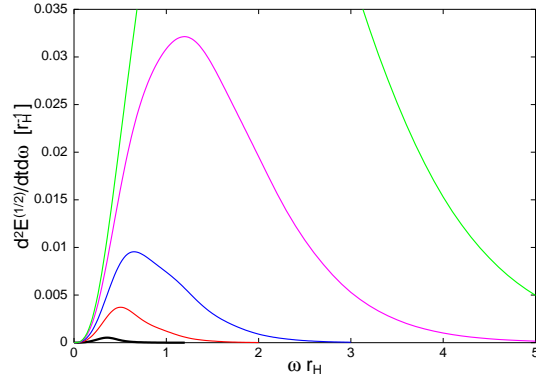


Fig. 1 – Energy emission rates for fermions on the brane or $n = 0, 1, 2, 4$ and 6 (from bottom to top) [7].

the formula

$$|\mathcal{A}(\omega)|^2 \equiv 1 - |\mathcal{R}(\omega)|^2 \equiv \frac{\mathcal{F}_{\text{horizon}}}{\mathcal{F}_{\text{infinity}}}, \quad (13)$$

where $\mathcal{R}(\omega)$ is the Reflection coefficient and \mathcal{F} the flux of energy towards the black hole. The task of solving the radial equation has been accomplished in the literature both analytically [18, 22] and numerically [23]. By studying the scattering problem in the aforementioned black-hole background, one may determine the value of the greybody factor, which when substituted in Eq. (8), can provide the radiation spectra of the emission of different species of fields on the brane. In Fig. 1, we depict the differential emission rate per unit time and frequency for fermions, in terms of the number of transverse spacelike dimensions n . The energy emission rate for fermions on the brane is greatly enhanced by the number of additional spacelike dimensions, and this holds also for scalar and gauge bosons as may be seen from the entries of Table 1, where the total emissivities have been normalised to the ones for $D = 4$ [23].

The situation in the case of the emission of Hawking radiation by a spherically-symmetric black hole is thus particularly simple. For a given mass, the only other parameter characterising the gravitational background is the number of additional spacelike dimensions n . Thus, comparing the predicted emission rates with the ob-

Table 1.

Total emissivities for brane-localised scalars, fermions and gauge bosons [23]

n	0	1	2	3	4	5	6	7
Scalars	1.0	8.94	36.0	99.8	222	429	749	1220
Fermions	1.0	14.2	59.5	162	352	664	1140	1830
G. Bosons	1.0	27.1	144	441	1020	2000	3530	5740

served ones, one may even hope to determine the value of n . The same number determines also the species of particles preferably emitted by a black hole: lower ($n = 0, 1$) and higher-dimensional ($n = 5, 6$) black holes prefer to emit scalars and gauge bosons, respectively, while black holes with $n = 2, 3, 4$ have a more ‘democratic’ type of spectrum [23].

In the case, however, of a higher-dimensional rotating black hole, that emits Hawking radiation, the situation changes considerably. The gravitational background around such a black hole is significantly more complicated and its line-element takes the form of the Myers-Perry solution [12]

$$ds^2 = \left(1 - \frac{\mu}{\Sigma r^{n-1}}\right) dt^2 + \frac{2a\mu \sin^2 \theta}{\Sigma r^{n-1}} dt d\varphi - \frac{\Sigma}{\Delta} dr^2 - \Sigma d\theta^2 - \left(r^2 + a^2 + \frac{a^2 \mu \sin^2 \theta}{\Sigma r^{n-1}}\right) \sin^2 \theta d\varphi^2 - r^2 \cos^2 \theta d\Omega_n^2, \quad (14)$$

where now

$$\Delta = r^2 + a^2 - \frac{\mu}{r^{n-1}}, \quad \Sigma = r^2 + a^2 \cos^2 \theta. \quad (15)$$

The above line-element describes the particular case of a ‘simply-rotating’ black hole, i.e. a black hole with only one non-vanishing angular momentum component – this is justified by the fact that the black hole has been created by particles that are localised on the brane and have a non-zero impact parameter only along a brane spacelike coordinate. The parameters μ and a are then associated to the black hole mass and angular momentum, respectively, through the relations

$$M_{BH} = \frac{(n+2)A_{2+n}}{16\pi G} \mu \quad \text{and} \quad J = \frac{2}{n+2} a M_{BH}, \quad (16)$$

where A_{2+n} is the area of a $(2+n)$ -dimensional unit sphere. The horizon radius is found by setting $\Delta(r_H) = 0$ and is found to be: $r_H^{n+1} = \mu/(1 + a_*^2)$, where we have defined the quantity $a_* \equiv a/r_H$.

We are again primarily interested in the radiation spectra emitted by the rotating black hole on the brane. The line-element that is seen by the brane-localised Standard Model fields follows again by fixing the values of the ‘‘extra’’ angular coordinates – in that case, the $d\Omega_n^2$ part of the metric (14) disappears while the remaining stays unaltered. A similar analysis, as in the case of a spherically-symmetric black hole, leads again to a master equation for the propagation of an arbitrary spin- s field on the brane background. For the particular line-element, this general equation decouples to a radial and angular part, given respectively by the following equations [7, 24]

$$\Delta^{-s} \frac{d}{dr} \left(\Delta^{s+1} \frac{dR_s}{dr} \right) + \left[\frac{K^2 - iKs\Delta'}{\Delta} + 4is\omega r + s(\Delta'' - 2) \delta_{s,|s|} - \Lambda_{sj}^m \right] R_s = 0 \quad (17)$$

and

$$\frac{1}{\sin \theta} \frac{d}{d\theta} \left(\sin \theta \frac{dS_{sj}^m}{d\theta} \right) + \left[-\frac{2ms \cot \theta}{\sin \theta} - \frac{m^2}{\sin^2 \theta} + a^2 \omega^2 \cos^2 \theta - 2a\omega s \cos \theta + s - s^2 \cot^2 \theta + \lambda_{sj} \right] S_{sj}^m(\theta) = 0, \quad (18)$$

where we have used the definitions

$$K = (r^2 + a^2)\omega - am, \quad \Lambda_{sj}^m = \lambda_{sj} + a^2\omega^2 - 2am\omega. \quad (19)$$

The angular eigenvalue λ_{sj} appearing in the angular equation does not exist in closed form. It may be computed either analytically, through a power series expansion in terms of $a\omega$ [25–27] or numerically [24, 28–30].

As in the case of spherically-symmetric black holes, one may compute the emission spectra for all species of particles propagating in the gravitational background of the rotating black hole on the brane. For this, we need the value of the Absorption Probability that can be found by solving the radial equation either analytically [31, 32] or numerically [24, 28–30, 33–35]. Then, the differential energy emission rate is given by the formula [14, 15, 36]

$$\frac{d^2 E}{dt d\omega} = \frac{1}{2\pi} \sum_{j,m} \frac{|\mathcal{A}(\omega)|^2 \omega}{\exp(\tilde{\omega}/T_H) \mp 1}, \quad (20)$$

while the temperature and rotation velocity of the black hole are given by

$$T_H = \frac{(n+1) + (n-1)a_*^2}{4\pi(1+a_*^2)r_H}, \quad \Omega_H = \frac{a}{(r_H^2 + a^2)}, \quad (21)$$

and $\tilde{\omega} = \omega - m\Omega_H$.

However, unlike the spherically-symmetric case, the gravitational background, and consequently the radiation spectra, now depend on two topological parameters, the angular-momentum parameter a_* of the black hole and the number of additional spacelike dimensions n . We therefore need to explore the effect that both these parameters have on the radiation spectra. It turns out that both enhance the energy emission rate: the enhancement factor is of order $\mathcal{O}(10)$ in terms of a_* and of order $\mathcal{O}(100)$ in terms of n . In Fig. 2, we depict the energy emission rates, for brane-localised scalars and gauge bosons, in terms of these two parameters, that clearly exhibit the aforementioned dependence.

It is actually the similar effect that the angular momentum of the black hole and the number of extra spacelike dimensions have on the radiation spectra that poses the biggest problem in our effort to determine the value of each. To lift this degeneracy, we need an observable that will depend rather strongly on only one of them and at the same time be almost insensitive to the value of the second. The solution is

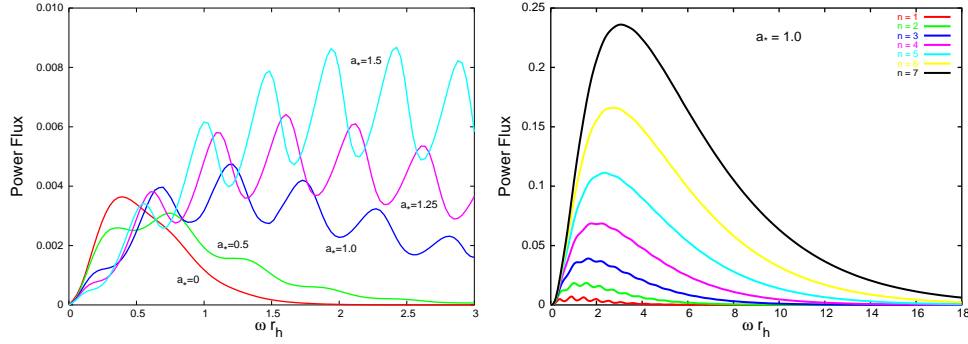


Fig. 2 – Energy emission rates for brane-localised scalar fields in terms of the angular parameter (left plot) [29] and gauge bosons in terms of the number of extra dimensions (right plot) [24].

provided by the angular distribution that characterises the radiation spectra emitted by a rotating black hole. This information is encoded in the angular equation (18) satisfied by the spin-weighted spheroidal harmonics $S_{s\ell}^m$. This equation was solved numerically [24, 29, 30] and it was found that two factors determine the angular distribution: first, the centrifugal force causes all emitted particles with intermediate and high frequency to be emitted along the equatorial plane; and second, the spin-rotation coupling forces the emission of particles with non-vanishing spin and low frequency to be aligned either in parallel or antiparallel with the rotation axis (see Fig. 3).

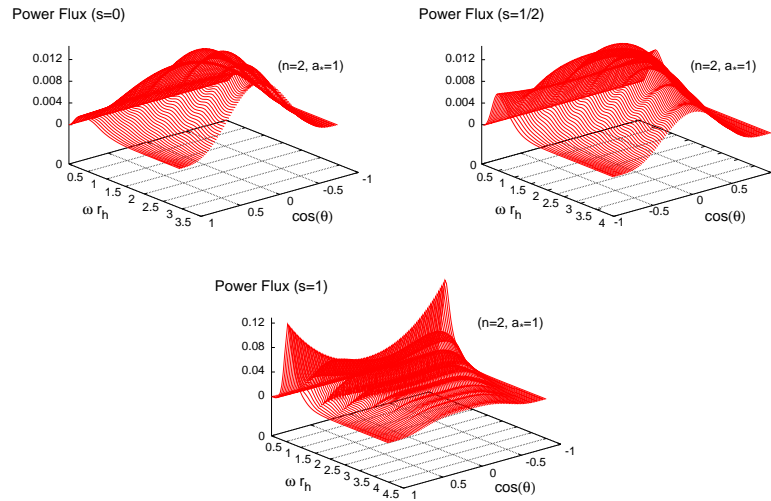


Fig. 3 – The angular distribution of (a) scalars (upper left plot) [29], (b) fermions (upper right plot) [24], and (c) gauge bosons (lower plot) [30] emitted on the brane by a 6D black hole with $a_* = 1$.

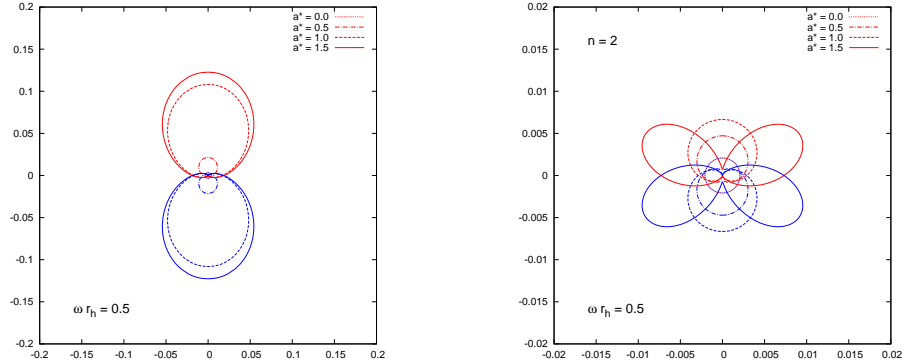


Fig. 4 – A polar plot with the angular distribution of (a) gauge bosons (left plot), and (b) fermions (right plot) emitted by a 6-dimensional, rotating black hole on the brane in the energy channel $\omega r_H = 0.5$ [37]. Red and blue curves correspond to positive and negative helicity particles.

The latter effect is more prominent the larger the value of the spin is. As a result, we expect the alignment of low-energy gauge bosons to be the best indicator of the orientation of the rotation axis of the black hole [24, 37]. This is indeed evident from Fig. 4(a), where gauge bosons emitted in the energy channel $\omega r_H = 0.5$ are perfectly aligned along the rotation axis independently of the value of the angular momentum of the black hole – note that the rotation axis runs vertically along the line $x = 0$. Once the orientation of the rotation axis is found, the angular distribution of low-energy fermions, that is sensitive to the value of a_* (see Fig. 4(b)) [30, 37, 38], can now be used in order to determine the value of the angular-momentum of the black hole itself. As was shown in [37], the above behaviour remains unaffected as the number of transverse-to-the-brane extra spacelike dimensions varies.

4. DECAY OF HIGHER-DIMENSIONAL BLACK HOLES IN THE BULK

Another important question is that of the energy balance between the emission on the brane and that in the bulk. The bulk emission can never be detected by a brane observer, and any amount of energy emitted in this channel will be missing energy in a particle experiment. However, one should know how much energy is spent on the bulk emission as that determines the amount of energy left for emission on the brane. According to the assumptions of the brane-world models, Standard-Model particles are restricted to live on the brane and are therefore emitted, via the process of Hawking radiation, only on the brane. In the bulk, we may have only particles that do not carry quantum numbers under the Standard-Model group, namely gravitons and possibly scalar fields. We thus need to investigate the emission of both these species of fields in the bulk during the evaporation process of the black hole.

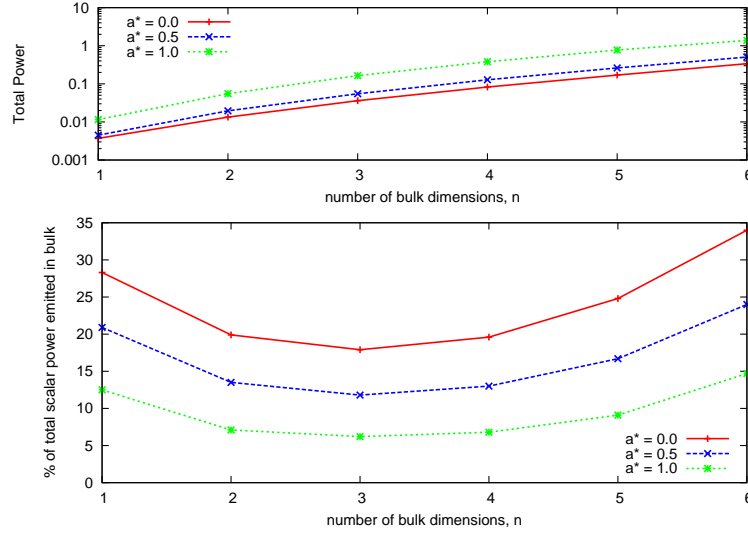


Fig. 5 – Total power emitted by a rotating black hole in the scalar channel (upper plot) and the % of this power emitted in the bulk (lower plot) [42].

The case of scalar fields propagating in the background of a higher-dimensional black hole is the easiest one to study. Their equation of motion can be decoupled, for both the spherically-symmetric and rotating phase, into a radial and an angular part. The scattering problem may be solved along the same lines as on the brane and the greybody factors are found both analytically [18, 39, 40] and numerically [23, 41, 42]. For the spherically-symmetric phase, the relative emissivity, i.e. the ratio of the total energy emitted by the black hole per unit time in the bulk over the one on the brane, is found to depend only on the number of extra dimensions: its value remains always below unity as n varies from 1 to 7, however, the emission in the two channels becomes comparable for the highest values of n [23]. In [42], the emission of scalar fields during the spin-down phase was studied, and it was demonstrated that the total energy output of the black hole increases with the angular momentum – however, the relative emissivity reduces further, compared to the Schwarzschild phase (see Fig. 5), due to the fact that the enhancement of the greybody factor in the bulk is smaller than the one on the brane. As a result, the brane dominance persists even in the spin-down phase, at least in the scalar channel of emission.

To settle this question, one would need to consider also the emission of gravitons in the bulk. This part of the study is not yet complete: while the field equations for gravitational perturbations in a higher-dimensional, spherically-symmetric background have been derived [43], the ones in an axially-symmetric spacetime remain largely unknown. For the Schwarzschild phase, the existing analyses, analyti-

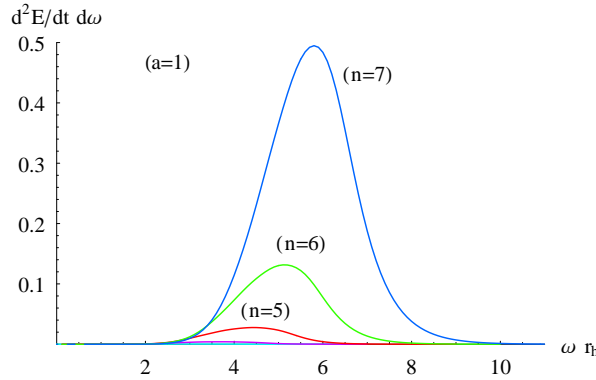


Fig. 6 – Energy emission rate for tensor-type gravitons in the bulk during the rotating phase [53].

cal [44, 45] as well as numerical [46, 47] ones, have revealed that gravitons carry an increasingly large amount of energy in the bulk as the number n of the extra dimensions increases, too. Although the enhancement factor reaches the value of 10^6 , the overall energy carried into the bulk, compared to the one released in total in the form of SM brane fields, is still sub-dominant.

The above results offer further support to an early work [48] where such a claim was made. Nevertheless, since special cases where the bulk channel may dominate over the brane one are known [49, 50], we still need to clarify the situation for the gravity emission in the bulk during the rotating phase. However, the graviton equations are known in only some very particular cases, one of them being the case of the simply rotating black hole where the field equations for tensor only gravitational perturbations have been derived [51]. In this case, the radiation spectra have the form of Fig. 6, where we depict the tensor-type graviton spectrum in terms of n for a black hole with $a_* = 1$ [52, 53]. In [53], an estimate of the total percentage of energy going into the bulk tensor graviton channel, compared to the one going into the scalar channel, was made: for the indicative case of $a_* = 1$, the energy carried by gravitons into the bulk is negligible for low values of n , but it reaches the value of 25% for $n = 5$. In order to draw a conclusive answer on whether the gravitons may tilt the bulk-to-brane energy balance towards the bulk during the rotating phase, we clearly need to include in our study the vector and scalar gravitational perturbations, too.

Finally, let us report results from a related work where the effect of the mass of the emitted scalar particles on the energy emission rates both in the bulk and on the brane, and consequently on the relative emissivity between the two channels, was studied [54]. In Fig. 7, we depict the indicative cases of the emission of scalar fields on the brane with $m_\Phi = 0, 0.4, 0.8$ by a black hole with angular-momentum parameter $a_* = 1$ and for three different values of n . As expected, the energy emission

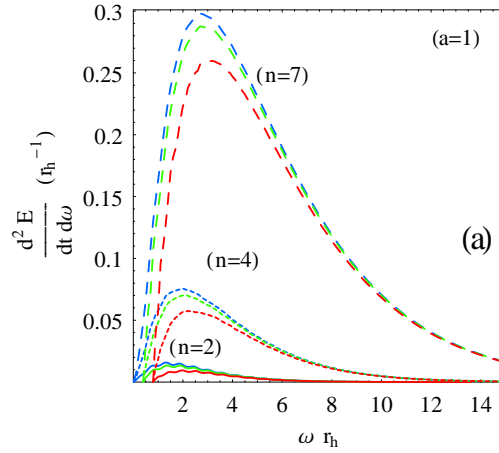


Fig. 7 – Energy emission rates for massive scalar fields on the brane during the rotating phase for $m_\Phi = 0, 0.4, 0.8$ (from top to bottom in each set of curves) [54].

rate is reduced as the mass of the emitted field increases – clearly, the emission of a massive field demands more energy to be spent by the black hole, and thus it is less likely to happen. However, this suppression is more prominent for the brane channel than for the bulk one and this gives, at particular cases, a considerable boost to the bulk-over-brane energy ratio: e.g. for $m_\Phi = 0.8$, the bulk-to-brane relative emissivity can be increased by 34% if $a_* = 0.5$ and $n = 2$ – as either a_* or n increases, the enhancement takes smaller but still significant values.

As is only natural, many experiments looking for physics beyond the Standard Model have included searches for miniature black holes in their research programs. Until now, no such effect has been observed with the most recent limits being the ones derived by the Large Hadron Collider at CERN. Both CMS [55] and ATLAS [56] collaborations have released data from proton-proton collisions corresponding to center-of-mass energy of 7 TeV and integrated luminosities of 35 pb^{-1} and 36 pb^{-1} , respectively. The CMS collaboration saw no excess above the predicted QCD background, and therefore concluded that, at 95% CL, no black holes exist with a minimum mass of (3.5-4.5) TeV in models with $n = 2, 4, 6$ and $M_* = (1.5 - 3.0) \text{ TeV}$. On the other hand, the ATLAS collaboration, in the absence again of any events, excluded the existence of black holes in models with $n = 6$ and $M_* = (0.75 - 3.67) \text{ TeV}$. Nevertheless, as was discussed in section 2, the window of energy for the creation of black holes might not be open yet as the minimum required energy for such an effect is around 8 TeV. Therefore, we should still wait for LHC to reach the ultimate goal of 14 TeV before drawing our final conclusions.

5. CONCLUSIONS

During the last decade, the topic of the Hawking radiation emission spectra from higher-dimensional, miniature black holes has undergone an intensive research. In this talk, I have addressed two questions that still remain open: the possibility of drawing information on the fundamental parameters of the higher-dimensional black-hole background, namely the dimensionality of spacetime in which it was formed and the value and orientation of angular momentum with which it was formed, and the determination of the relative bulk-over-brane energy emissivity. For the former question, we have proposed a solution related to the angular variation of the emitted radiation that is characteristic of the emission from a rotating black hole. For the latter question, the existing results in the literature point to the conclusion that the brane channel is dominant over the bulk one, during both the spin-down and the Schwarzschild phase, nevertheless the study of the radiation spectra for all types of gravitational perturbations is not complete yet.

Acknowledgements. I am grateful to my collaborators (S. Dolan, M. Casals, R. Konoplya, H. Kodama, N. Pappas, E. Winstanley and A. Zhidenko) for our enjoyable and fruitful collaborations. I would also like to thank the organisers of the SEENET-MTP Workshop BW2011 “Particle Physics from TeV to Planck Scale” for their kind invitation to present this talk and for their support.

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